How and Why E Works !

or

Hand-optimized Z80 assembly code

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Introduction

E: Successful high-performance equational theorem prover for full clausal logic

Based on paramodulation and rewriting (modified superposition calculus)

Fully automatic (no user interaction during run time)

Emphasis on search control heuristics

Available in source form (compiles on all known current UNIX versions) at `http://www4.informatik.tu-muenchen.de/~schulz/WORK/eprover.html`

Current stable release: E 0.6 Kanchanjangha

What you all want to know: Releases are named after tea gardens

Overview

Calculus

Search organization

Implementation

Search control

Lessons learned

– Calculus –

Superposition (with literal selection)

Saturating calculus, works on (sets of) clauses in equational representation

Generating inferences:

- Superposition
- Equality resolution
- Equality factoring

Most important inference ($>$ 95% of new clauses): Superposition (paramodulation constrained by simplification ordering)

Overlap (only) maximal terms of maximal positive literals into maximal terms of (maximal xor selected) literals

Literal selection

Completeness-maintaining refinement of superposition/resolution

(Informal) idea: View clause as set of conditional equations:

- $l_1 = r_1 \vee l_2 = r_2 \vee s_1 \neq t_1 \vee s_2 \neq t_2$
  $\hat{=} \{l_1 = r_1 \leftarrow l_2 \neq r_2 \wedge s_1 = t_1 \wedge s_2 = t_2,$
  $l_2 = r_2 \leftarrow l_1 \neq r_1 \wedge s_1 = t_1 \wedge s_2 = t_2\}$
- We do not know which negative condition (i.e. positive literal) we need to solve
- But: We need to solve all positive conditions (i.e. negative literals)
- Ergo: We can pick one arbitrary negative literal as the first one to solve

More generally: If there is at least one negative literal in a clause, inferences can be restricted to an arbitrary subset of **selected** literals containing at least one negative literal (Don't-care-nondeterminism)

We do not need to paramodulate from a clause with selected literals at all

Reducing redundancy

Redundancy criteria (implemented as contracting inference rules) allow us to simplify or eliminate clauses

Useful mechanisms:

- Eliminating duplicate/trivial literals: $\{u = v \vee v = u\} \Longrightarrow \{u = v\}$ and $\{u \neq u\} \Longrightarrow \{\square\}$
- Rewriting (using oriented unit-clauses): $\{a = b, f(a) = c\} \Longrightarrow \{a = b, f(b) = c\}$ (if $a > b$)
- Tautology elimination (only partially possible): $\{a = b \vee a \neq b\} \Longrightarrow \{\}$
- Subsumption (Powerful, but often expensive): $\{f(x) = f(y), f(a) = f(b) \vee s = t\} \Longrightarrow \{f(x) = f(y)\}$
- Simplify-reflect (generalized equational unit-resolution): $\{g(x) = g(y), f(g(a)) \neq f(g(b)) \vee s = t\} \Longrightarrow \{g(x) = g(y), s = t\}$

Important for efficiency: Apply contracting rules early and often

– Search Organization –

Aims of a Proof Procedure

Fast (do as little as possible)

Lazy (delay work as much as possible)

Minimal (avoid redundant steps)

Complete (do as much as necessary)

Intelligent (allow good use of guiding heuristics)

Our Solution (DISCOUNT loop)

Variant of given-clause algorithm

Strict separation of active and passive clauses

Invariants of active (or processed) clause set **P**:

- All clauses maximally simplified (with respect to **P**)
- Contains no clauses that are (provably) redundant within **P**
- All generating inferences between clauses in **P** performed

Passive or unprocessed clauses (**U**)

- Sorted by one or more heuristic evaluations (best-first access)
- Clauses simplified once (at creation) with then current **P**
- No expensive redundancy checks (e.g. full tautology, non-unit subsumption)

Initial state: **P** empty, axioms (and goals) in **U**

DISCOUNT loop (continued)

while($\mathbf{U} \neq \emptyset$)
  $\mathbf{g}$:= remove_best($\mathbf{U}$)
  simplify $\mathbf{g}$ with $\mathbf{P}$
  if($\mathbf{g} = \square$) success; end;endif
  if($\mathbf{g}$ is not redundant in $\mathbf{P}$)
    simplify $\mathbf{P}$ with $\mathbf{g}$ and interreduce $\mathbf{P} \cup \{\mathbf{g}\}$
    (remove redundant clauses, move critically affected clauses into $\mathbf{U}$, kill offspring)
    $\mathbf{T}$:= generate($\mathbf{P}$,$\mathbf{g}$)
    $\mathbf{P} := \mathbf{P} \cup \{\mathbf{g}\}$
    simplify $\mathbf{T}$ with $\mathbf{P}$
    $\mathbf{U} := \mathbf{U} \cup \mathbf{T}$
  endif
endwhile
Failure, initial clause set is satisfiable

– Implementation –

Some Important Aspects

Term representation

Indexing

Subsumption

Memory management

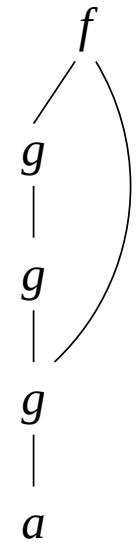
Term Representation

Terms in E are perfectly shared – no term ever is represented twice unless the two copies are distinguished by the calculus (saves 90%–99.9% of all term nodes)

Heavy terms: Precomputed weights, normal form time stamps, reducibility status, . . .

Rewriting is performed directly on the shared representation:

$$f(g(g(g(a), g(a)))) \Longrightarrow_{g(x)\to x} f(a, a) \qquad \hat{=}$$



$$\Longrightarrow$$

Example: Shared Terms & Forward Rewriting

Situation (Simplifying newly generated clauses):

- Given: System **R** of rewrite rules (and equations)
- Given: (Large) set **T** of new clause (generated by superposition) and (larger) set **U** of old unprocessed clauses
- Aim: Find **R**-normal forms of all terms in **T**

Benefits of shared rewriting:

- Typical: Many common terms in **T**
  $\Rightarrow$ Each term only needs to be considered once
- Typical: Many common terms in **T** and **U**
  $\Rightarrow$ Many terms are already in normal form and are recognized as such
- Many remaining common terms are in normal form with respect to earlier **R**
  $\Rightarrow$ Age-constraints speed up matching (see below)
- Instances of common terms in **U** are rewritten for free

Indexing in E

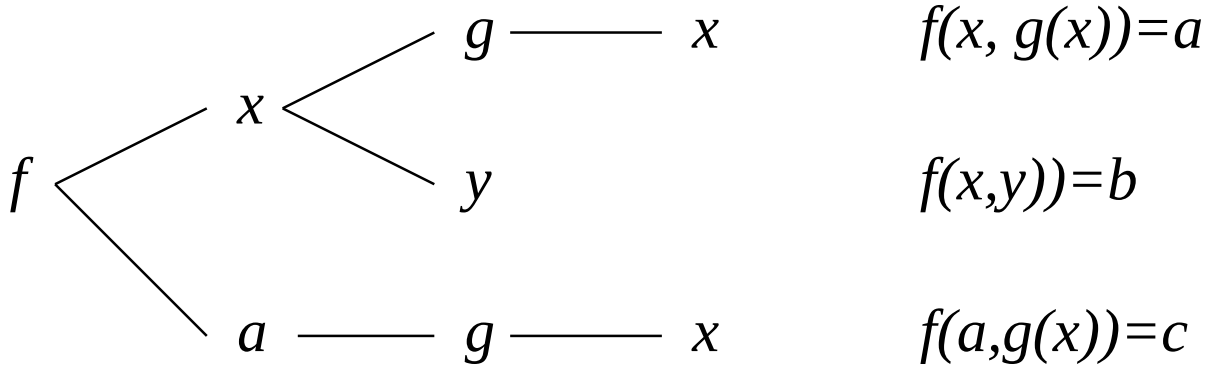
Indexing: Speed up clause-set/clause or clause-set/clause-set operations

E currently indexes only processed unit clauses

Index used for rewriting, unit-subsumption, simplify-reflect

Index type: Perfect discrimination tree

– Consider term as linear word
– Branch on differences in words



Constrained Perfect Discrimination Trees

Aim: Speed up matching (in particular for rewriting)

Size constraints:

– A term can only match terms that are of at least equal size

Age constraints:

– Reducibility relation becomes monotonically stronger over time
– Terms in normal form with respect to clauses at time t can at most be rewritten by younger clauses

How to use this:

– Annotate branches in discrimination tree with age of youngest indexed clause and size of smallest indexed left hand side
– Consider only branches with satisfiable constraints during matching

Subsumption

Unit-subsumption:

- Index on maximal terms of candiate clauses
- Sequential search for matching right hand side
- Only unorientable clauses are candidates (otherwise: Rewriting possible)

Non-unit-subsumption:

- Sequential search through candidate clauses
- Pre-tests (literal numbers, symbol count, individual literal matches with literal symbol count) for filtering
- Standard backtracking algorithm (modulo symmetry of equations)
- Rather slow, but application kept to a minimum by proof procedure
  $\Rightarrow$ Overall cost is acceptable

Memory management

Keeping memory manageable:

- Prover keeps track of approximate memory use (term cells, literals, clauses)
- If memory reaches predefined limit, 50% of clauses are discarded
- Discarding: Simulate future clause selection strategy, delete the clauses that would be selected last

Managing memory:

- Intermediate abstract layer above C standard `malloc`
- Keeps free lists for memory blocks of relevant sizes, reuse freed blocks
- Complex operations (reorganization of free memory) left to C standard library
- Optionally: Full logging of memory operations and/or counting of allocated/freed blocks

– Search Control –

Don't-care-Nondeterminism ≡ Chances for Heuristics

Important choice points for E:

- Simplification ordering
- Clause selection
- Literal selection

Other choice points:

- Choice of rewrite relation (usually strongest, don't care which normal form)
- Application of rewrite relation to terms (leftmost-innermost, strongly suggested by shared terms)

Simplification Orderings

Implemented: Knuth-Bendix-Orderings, Lexicographic Path Orderings

Precedence: Fully user defined or simple algorithms

- **Sorted by arity** (higher arity $\longrightarrow$ larger)
- **Sorted by arity, but unary first**
- Sorted by inverse arity
- Sorted by frequency of appearance in axioms
- . . .

Weights for KBO: Similar simple algorithms (constant weights (optionally weight 0 for maximal symbol), arity, position in precedence . . .)

No good automatic selection of orderings yet – auto mode switches between two simple KBO schemes

Clause selection

Most important choice point

Probably also hardest chocice (find best clause among millions)

Implementation in E: Multiple priority queues sorted by heuristic evaluation and strategy-defined priority

Selection in weighted round-robin-scheme (generalizes pick-given ratio)

Example: 8*Refinedweight(PreferGoals,1,2,2,3,0.8),
8*Refinedweight(PreferNonGoals,2,1,2,3,0.8),
1*Clauseweight(ConstPrio,1,1,0.7),
1*FIFOWeight(ByNegLitDist)

Auto-mode: Select between strategies based on problem features (Unit/Horn/General, Pure/Some/no equality, number of clauses, . . .)

Auto-mode automatically generated from test results

Literal Selection

Problem: Which literals should be selected for inferences in a clause?

Ideas:

- Select hard literals first (if we cannot solve this, the clause is useless)
- Select small literals (fewer possible overlaps)
- Select ground literals (no instantiation, most unit-overlaps eleminated by rewriting)
- Propagate inference literals to children clauses (inheritance)

Problem: Should we always select literals if possible?

- Only select if no unique maximal literal exists
- Do not select in conditional rewrite rules

Surprisingly successful: Additional selection of maximal positive literals

See E source code for large number of things we have tried. . .

– Lessons Learned –

Development, Bondage, and Discipline

Develop around a small number of strong central ideas

- Shared terms and shared rewriting
- DISCOUNT loop
- Round-robin clause selection

Later add-ons can destroy your code base!

- Only add features that fit well into your framework
- Minimize ad-hoc hacks
- Note: One often cannot judge the value of a technique from a prototypical implementation anyways

Don't hesitate to throw code away

Optimization

Use profiling tools to find hot spots (guessing is bad!)

– DISCOUNT: Unification $<1\%$, list management up to 50%

Don't over-optimize (it may backfire anyways)

Optimize overall performance, not (perhaps) useless operations

– (Perhaps avoid) Repeated simplification of unprocessed clauses
– (Perhaps avoid) Full redundancy tests on unprocessed clauses

$O()$ is important, k is not (as long as it is small enough)

Memory is more of a bottleneck than CPU

Search Control

If inference speed is sufficient, search control is crucial for performance

Intuition is often wrong – experimental evaluation is very important

Allow for flexible use of heuristics at important choice points

Experiences from other provers do not always apply

Distrust authority and legends

- Especially older folklore does not always hold for current hardware/software
- 5 is not the optimal pick-given ratio

Assorted Topics – Conclusion

Perfectly shared terms with shared rewriting are a success

- Useful in forward and backward rewriting
- Gives many of the benefits of more involved indexing techniques
- Saves memory (no. of terms cells $\approx$ no. of literals)
- But: Using 50% of the potential gain made me loose 50% of my hair. . .

Don't hesitate to ~~steal~~ accept good ideas from other people or provers

Future work . . .
. . . will we revealed after CASC-18 **;-)**