

# E 1.0x

## User Manual

–preliminary version–

Stephan Schulz

October 22, 2008

### Abstract

E is an equational theorem prover for full clausal logic, based on superposition and rewriting. In this *very preliminary* manual we first give a short introduction for impatient new users, and then cover calculus, control, options and input/output of the prover in some more detail.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Getting Started</b>	<b>2</b>
<b>3</b>	<b>Calculus and Proof Procedure</b>	<b>3</b>
3.1	Calculus . . . . .	3
3.2	Proof Procedure . . . . .	7
<b>4</b>	<b>Usage</b>	<b>9</b>
4.1	Search Control Heuristics . . . . .	9
4.1.1	Priority functions . . . . .	9
4.1.2	Generic Weight Functions . . . . .	12
4.1.3	Clause Evaluation Functions . . . . .	13
4.1.4	Heuristics . . . . .	14
4.2	Term Orderings . . . . .	14
4.2.1	Precedence Generation Schemes . . . . .	16
4.2.2	Weight Generation Schemes . . . . .	17
4.3	Literal Selection Strategies . . . . .	18
4.4	The Watchlist Feature . . . . .	20
4.5	Learning Clause Evaluation Functions . . . . .	21
4.5.1	Creating Knowledge Bases . . . . .	21

4.5.2	Populating Knowledge Bases . . . . .	21
4.5.3	Using Learned Knowledge . . . . .	21
4.6	Other Options . . . . .	22
<b>5</b>	<b>Input Language</b>	<b>22</b>
5.1	LOP . . . . .	22
5.2	TPTP Format . . . . .	22
<b>6</b>	<b>Output...or how to interpret what you see</b>	<b>23</b>
6.1	The Bare Essentials . . . . .	23
6.2	Impressing your Friends . . . . .	25
6.3	Detailed Reporting . . . . .	25
6.4	Requesting Specific Results . . . . .	25
<b>A</b>	<b>License</b>	<b>27</b>

## 1 Introduction

This is a short and currently very sketchy documentation to the E equational theorem prover. E is an purely equational theorem prover for first-order logic with equality. It is based on paramodulation and rewriting. This means that E reads a set of formulas and/or clauses and saturates it by systematically applying a number of inference rules until either all possible inferences have been performed or until the empty clause has been derived, i.e. the clause set has been found to be unsatisfiable and thus a conjecture has been proved.

E is still a moving target, but most recent releases have been quite stable, and the prover is being used productively by several independent groups of people. This manual should enable you to experiment with the prover and to use some of its more advanced features.

The manual assumes a working knowledge of refutational theorem proving, which can be gained from e.g. [CL73]. For a short description of E including performance data, see [Sch04]. A more detailed description has been published as [Sch02]. Most papers on E and much more information is available at or a few hops away from the E home page, <http://www.e prover.org>.

Some other provers have influenced the design of E and may be referenced in the course of this manual. These include SETHEO [MIL<sup>+</sup>97], Otter [McC94, MW97], SPASS [WGR96, WAB<sup>+</sup>99], DISCOUNT [DKS97], Waldmeister [HBF96, HJL99] and Vampire [RV02, RV01].

## 2 Getting Started

Installation of E should be straightforward. The file **README** in the main directory of the distribution contains the necessary information. After building, you will find the stand-alone executable **E/PROVER/eprover**.

E is controlled by a very wide range of parameters. However, if you do not want to bother with the details, you can leave configuration for a problem to the prover. To use this feature, use the following command line options:

<code>-xAuto</code>	Select a literal selection strategy and a selection heuristic automatically (based on problem features).
<code>-tAuto</code>	Select a term ordering automatically.
<code>--memory-limit=xx</code>	Tell the prover how much memory (measured in MB) to use at most. In automatic mode E will optimize its behaviour for this amount (20 MB will work, 64 MB is reasonable, 192 MB is what I use. <i>More is better</i> <sup>1</sup> , but if you go over your physical memory, you will probably experience <i>very</i> heavy swapping.).

*Example:* If you happen to have a workstation with 64 MB RAM<sup>2</sup>, the following command is reasonable:

```
eprover -xAuto -tAuto --memory-limit=48 PUZ031-1+rm_eq_rstfp.lop
```

This documentation will probably lag behind the development of the latest version of the prover for quite some time. To find out more about the options available, type `eprover --help` (or consult the source code included with the distribution).

### 3 Calculus and Proof Procedure

E is a purely equational theorem prover, based on ordered paramodulation and rewriting. As such, it implements an instance of the superposition calculus described in [BG94]. We have extended the calculus with some stronger contraction rules and more general approach to literal selection. The proof procedure is a variant of the *given-clause* algorithm.

#### 3.1 Calculus

$Term(F, V)$  denotes the set of (first order) *terms* over a finite set of function symbols  $F$  (with associated arities) and an enumerable set of variables  $V$ . We write  $t|_p$  to denote the subterm of  $t$  at a position  $p$  and write  $t[p \leftarrow t']$  to denote  $t$  with  $t|_p$  replaced by  $t'$ . An equation  $s \simeq t$  is an (implicitly symmetrical) pair of terms. A positive literal is an equation  $s \simeq t$ , a negative literal is a

<sup>1</sup>Emphasis added for E 0.7 and up, which globally cache rewrite steps.

<sup>2</sup>Yes, this is outdated. If it still applies to you, get a new computer! It will still work ok, though.

negated equation  $s \not\approx t$ . We write  $s \simeq t$  to denote an arbitrary literal<sup>3</sup>. Literals can be represented as multi-sets of multi-sets of terms, with  $s \simeq t$  represented as  $\{\{s\}, \{t\}\}$  and  $s \not\approx t$  represented as  $\{\{s, t\}\}$ . A *ground reduction ordering*  $>$  is a Noetherian partial ordering that is stable w.r.t. the term structure and substitutions and total on ground terms.  $>$  can be extended to an ordering  $>_l$  on literals by comparing the multi-set representation of literals with  $\gggg$  (the multi-set-multi-set extension of  $>$ ).

Clauses are multi-sets of literals. They are usually represented as disjunctions of literals,  $s_1 \simeq t_1 \vee s_2 \simeq t_2 \dots \vee s_n \simeq t_n$ . We write  $Clauses(F, P, V)$  to denote the set of all clauses with function symbols  $F$ , predicate symbols  $P$  and variable  $V$ . If  $\mathcal{C}$  is a clause, we denote the (multi-)set of positive literals in  $\mathcal{C}$  by  $\mathcal{C}^+$  and the (multi-)set of negative literals in  $\mathcal{C}$  by  $\mathcal{C}^-$ .

The introduction of an extended notion of *literal selection* has improved the performance of E significantly. The necessary concepts are explained in the following.

**Definition 3.1 (Selection functions)**

$sel : Clauses(F, P, V) \rightarrow Clauses(F, P, V)$  is a *selection function*, if it has the following properties for all clauses  $\mathcal{C}$ :

- $sel(\mathcal{C}) \subseteq \mathcal{C}$ .
- If  $sel(\mathcal{C}) \cap \mathcal{C}^- = \emptyset$ , then  $sel(\mathcal{C}) = \emptyset$ .

We say that a literal  $\mathcal{L}$  is *selected* (with respect to a given selection function) in a clause  $\mathcal{C}$  if  $\mathcal{L} \in sel(\mathcal{C})$ . ◀

We will use two kinds of restrictions on deducing new clauses: One induced by ordering constraints and the other by selection functions. We combine these in the notion of *eligible literals*.

**Definition 3.2 (Eligible literals)**

Let  $\mathcal{C} = \mathcal{L} \vee \mathcal{R}$  be a clause, let  $\sigma$  be a substitution and let  $sel$  be a selection function.

- We say  $\sigma(\mathcal{L})$  is *eligible for resolution* if either
  - $sel(\mathcal{C}) = \emptyset$  and  $\sigma(\mathcal{L})$  is  $>_L$ -maximal in  $\sigma(\mathcal{C})$  or
  - $sel(\mathcal{C}) \neq \emptyset$  and  $\sigma(\mathcal{L})$  is  $>_L$ -maximal in  $\sigma(sel(\mathcal{C}) \cap \mathcal{C}^-)$  or
  - $sel(\mathcal{C}) \neq \emptyset$  and  $\sigma(\mathcal{L})$  is  $>_L$ -maximal in  $\sigma(sel(\mathcal{C}) \cap \mathcal{C}^+)$ .
- $\sigma(\mathcal{L})$  is *eligible for paramodulation* if  $\mathcal{L}$  is positive,  $sel(\mathcal{C}) = \emptyset$  and  $\sigma(\mathcal{L})$  is strictly  $>_L$ -maximal in  $\sigma(\mathcal{C})$ .

---

<sup>3</sup>Non-equational literals are encoded as equations or disequations  $P(t_1, \dots, t_n) \simeq \top$ . In this case, we treat predicate symbols as special function symbols that can only occur at the top-most positions and demand that atoms (terms formed with a top predicate symbol) cannot be unified with a first-order variable from  $V$ , i.e. we treat normal terms and predicate terms as two disjoint types.

◀

The calculus is represented in the form of inference rules. For convenience, we distinguish two types of inference rules. For *generating* inference rules, written with a single line separating preconditions and results, the result is added to the set of all clauses. For *contracting* inference rules, written with a double line, the result clauses are substituted for the clauses in the precondition. In the following,  $u, v, s$  and  $t$  are terms,  $\sigma$  is a substitution and  $R, S$  and  $T$  are (partial) clauses.  $p$  is a position in a term and  $\lambda$  is the empty or top-position. Different clauses are assumed to not share any common variables.

**Definition 3.3 (The inference system SP)**

Let  $>$  be a total simplification ordering (extended to orderings  $>_L$  and  $>_C$  on literals and clauses) and let  $sel$  be a selection function. The inference system **SP** consists of the following inference rules:

- *Equality Resolution:*

$$(ER) \frac{u \not\approx v \vee R}{\sigma(R)} \quad \text{if } \sigma = mgu(u, v) \text{ and } \sigma(u \not\approx v) \text{ is eligible for resolution.}$$

- *Superposition into negative literals:*

$$(SN) \frac{s \simeq t \vee S \quad u \not\approx v \vee R}{\sigma(u[p \leftarrow t] \not\approx v \vee S \vee R)} \quad \begin{array}{l} \text{if } \sigma = mgu(u|_p, s), \sigma(s) \not\prec \sigma(t), \sigma(u) \not\prec \sigma(v), \sigma(s \simeq t) \\ \text{is eligible for paramodulation, } \sigma(u \not\approx v) \text{ is eligible for} \\ \text{resolution, and } u|_p \notin V. \end{array}$$

- *Superposition into positive literals:*

$$(SP) \frac{s \simeq t \vee S \quad u \simeq v \vee R}{\sigma(u[p \leftarrow t] \simeq v \vee S \vee R)} \quad \begin{array}{l} \text{if } \sigma = mgu(u|_p, s), \sigma(s) \not\prec \sigma(t), \sigma(u) \not\prec \sigma(v), \sigma(s \simeq t) \\ \text{is eligible for paramodulation, } \sigma(u \not\approx v) \text{ is eligible for} \\ \text{resolution, and } u|_p \notin V. \end{array}$$

- *Equality factoring:*

$$(EF) \frac{s \simeq t \vee u \simeq v \vee R}{\sigma(t \not\approx v \vee u \simeq v \vee R)} \quad \text{if } \sigma = mgu(s, u), \sigma(s) \not\prec \sigma(t) \text{ and } \sigma(s \simeq t) \text{ eligible for paramodulation.}$$

- *Rewriting of negative literals:*

$$(RN) \frac{s \simeq t \quad u \not\approx v \vee R}{s \simeq t \quad u[p \leftarrow \sigma(t)] \not\approx v \vee R} \quad \text{if } u|_p = \sigma(s) \text{ and } \sigma(s) > \sigma(t).$$

- *Rewriting of positive literals*<sup>4</sup>:

$$(RP) \frac{s \simeq t \quad u \simeq v \vee R}{s \simeq t \quad u[p \leftarrow \sigma(t)] \simeq v \vee R}$$

if  $u|_p = \sigma(s)$ ,  $\sigma(s) > \sigma(t)$ , and if  $u \simeq v$  is not eligible for resolution or  $u \not\simeq v$  or  $p \neq \lambda$ .

- *Clause subsumption*:

$$(CS) \frac{C \quad \sigma(C \vee R)}{C}$$

where  $C$  and  $R$  are arbitrary (partial) clauses and  $\sigma$  is a substitution.

- *Equality subsumption*:

$$(ES) \frac{s \simeq t \quad u[p \leftarrow \sigma(s)] \simeq u[p \leftarrow \sigma(t)] \vee R}{s \simeq t}$$

- *Positive simplify-reflect*<sup>5</sup>:

$$(PS) \frac{s \simeq t \quad u[p \leftarrow \sigma(s)] \not\simeq u[p \leftarrow \sigma(t)] \vee R}{s \simeq t \quad R}$$

- *Negative simplify-reflect*

$$(NS) \frac{s \not\simeq t \quad \sigma(s) \simeq \sigma(t) \vee R}{s \simeq t \quad R}$$

- *Contextual (top level) simplify-reflect*

$$(CSR) \frac{\sigma(C \vee R \vee s \dot{=} t) \quad C \vee \overline{s \dot{=} t}}{\sigma(C \vee R) \quad C \vee s \dot{=} t} \quad \text{where } \overline{s \dot{=} t} \text{ is the negation of } s \dot{=} t \text{ and } \sigma \text{ is a substitution}$$

<sup>4</sup>A stronger version of (RP) is proven to maintain completeness for Unit and Horn problems and is generally believed to maintain completeness for the general case as well [Bac98]. However, the proof of completeness for the general case seems to be rather involved, as it requires a very different clause ordering than the one introduced [BG94], and we are not aware of any existing proof in the literature. The variant rule allows rewriting of maximal terms of maximal literals under certain circumstances:

$$(RP') \frac{s \simeq t \quad u \simeq v \vee R}{s \simeq t \quad u[p \leftarrow \sigma(t)] \simeq v \vee R}$$

if  $u|_p = \sigma(s)$ ,  $\sigma(s) > \sigma(t)$  and if  $u \simeq v$  is not eligible for resolution or  $u \not\simeq v$  or  $p \neq \lambda$  or  $\sigma$  is not a variable renaming.

This stronger rule is implemented successfully by both E and SPASS [Wei99].

<sup>5</sup>In practice, this rule is only applied if  $\sigma(s)$  and  $\sigma(t)$  are  $>$ -incomparable – in all other cases this rule is subsumed by (RN) and the deletion of resolved literals (DR).

- *Tautology deletion:*

$$(TD) \frac{C}{\quad} \quad \text{if } C \text{ is a tautology}^6.$$

- *Deletion of duplicate literals:*

$$(DD) \frac{s \simeq t \vee s \simeq t \vee R}{s \simeq t \vee R}$$

- *Deletion of resolved literals:*

$$(DR) \frac{s \not\simeq s \vee R}{R}$$

- *Destructive equality resolution:*

$$(DE) \frac{x \not\simeq y \vee R}{\sigma(R)} \quad \text{if } x, y \in V, \sigma = mgu(x, y)$$

We write  $\mathbf{SP}(N)$  to denote the set of all clauses that can be generated with one generating inference from  $I$  on a set of clauses  $N$ ,  $\mathcal{D}_{SP}$  to denote the set of all  $\mathbf{SP}$ -derivations, and  $\mathcal{D}_{\overline{SP}}$  to denote the set of all finite  $\mathbf{SP}$ -derivations. ◀

As  $\mathbf{SP}$  only removes clauses that are *composite* with respect to the remaining set of clauses, the calculus is complete. For the case of unit clauses, it degenerates into *unfailing* completion [BDP89] as implemented in DISCOUNT. E can also simulate the positive unit strategy for Horn clauses described in [Der91] using appropriate selection functions.

Contrary to e.g. SPASS, E does not implement special rules for non-equational literals or sort theories, as we expect this part to be taken care of by SETHEO in a later combined system. Instead, non-equation literals are encoded as equations and dealt with accordingly.

### 3.2 Proof Procedure

Fig. 1 shows a (slightly simplified) pseudocode sketch of the quite straightforward proof procedure of E<sup>7</sup>. The set of all clauses is split into two sets, a set

---

<sup>6</sup>This rule can only be implemented approximately, as the problem of recognizing tautologies is only semi-decidable in equational logic. Current versions of E try to detect tautologies by checking if the ground-completed negative literals imply at least one of the positive literals, as suggested in [NN93].

<sup>7</sup>Note that the proof procedure has been further simplified for version 0.8 and up. The pseudo-code describes the current version.

```

# Input:  Axioms in U, P is empty
while U  $\neq$   $\emptyset$  begin
  c := select(U)
  U := U \ c
  # Apply (RN), (RP), (NS), (PS), (CSR), (DR), (DD), (DE)
  simplify(c,P)
  # Apply (CS), (ES), (TD)
  if c is trivial or subsumed by P then
    # delete(c)
  else if c is the empty clause then
    # Success:  Proof found
    stop
  else
    T :=  $\emptyset$  # Temporary clause set
    foreach p  $\in$  P do
      if c simplifies p
        P := P \ p
        T := T  $\cup$  p
      done
    end
    T := T  $\cup$  e-resolvents(c) # (ER)
    T := T  $\cup$  e-factors(c) # (EF)
    T := T  $\cup$  paramodulants(c,P) # (SN), (SP)
    foreach p  $\in$  T do
      # Apply efficiently implemented subset of (RN),
      # (RP), (NS), (PS), (CSR), (DR), (DD), (DE)
      p := cheap_simplify(p, P)
      # Apply (TD) or efficient approximation of it
      if p is trivial
        delete(p)
      else
        U := U  $\cup$  cheap_simplify(p, P)
      fi
    end
  fi
end
# Failure:  Initial U is satisfiable, P describes model

```

Figure 1: Main proof procedure of E



P of *processed* clauses and a set U of *unprocessed* clauses. Initially, all input clauses are in U, and P is empty. The algorithm selects a new clause from U, simplifies it w.r.t. to P, then uses it to back-simplify the clauses in P in turn. It then performs equality factoring, equality resolution and superposition between the selected clause and the set of processed clauses. The generated clauses are added to the set of unprocessed clauses. The process stops when the empty clause is derived or no further inferences are possible.

The proof search is controlled by three major parameters: The term ordering (described in section 4.2), the literal selection function, and the order in which the **select** operation selects the next clause to process.

E implements two different classes of term orderings, lexicographic term orderings and Knuth-Bendix orderings. A given ordering is determined by instantiating one of the classes with a variety of parameters (described in section 4.2).

Literal selection currently is done according to one of more than 50 predefined functions. Section 4.3 describes this feature.

Clause selection is determined by a heuristic evaluation function, which conceptually sets up a set of priority queues and a weighted round robin scheme that determines from which queue the next clause is to be picked. The order within each queue is determined by a priority function (which partitions the set of unprocessed clauses into one or more subsets) and a heuristic evaluation function, which assigns a numerical rating to each clause. Section 4.1 describes the user interface to this mechanism.

## 4 Usage

### 4.1 Search Control Heuristics

Search control heuristics define the order in which the prover considers newly generated clauses. A heuristic is defined by a set of *clause evaluation functions* and a selection scheme which defines how many clauses are selected according to each evaluation function. A clause evaluation function consists of a *priority function* and an instance of a generic *weight function*.

#### 4.1.1 Priority functions

Priority functions define a partition on the set of clauses. A single clause evaluation consists of a priority (which is the first selection criteria) and an evaluation. Priorities are usually *not* suitable to encode heuristical control knowledge, but rather are used to express certain elements of a search strategy, or to restrict the effect of heuristic evaluation functions to certain classes of clauses. It is quite trivial to add a new priority function to E, so at any time there probably exist a few not yet documented here.

Syntactically, a large subset of currently available priority functions is described by the following rule:

```
<prio-fun> ::= PreferGroundGoals ||
```

```

PreferUnitGroundGoals ||
PreferGround ||
PreferNonGround ||
PreferProcessed ||
PreferNew ||
PreferGoals ||
PreferNonGoals ||
PreferUnits ||
PreferNonUnits ||
PreferHorn ||
PreferNonHorn ||
ConstPrio ||
ByLiteralNumber ||
ByDerivationDepth ||
ByDerivationSize ||
ByNegLitDist ||
ByGoalDifficulty ||
SimulateSOS||
PreferHorn||
PreferNonHorn||
PreferUnitAndNonEq||
DeferNonUnitMaxEq||
ByCreationDate||
PreferWatchlist||
DeferWatchlist

```

The priority functions are interpreted as follows:

**PreferGroundGoals:** Always prefer ground goals (all negative clauses without variables), do not differentiate between all other clauses.

**PreferUnitGroundGoals:** Prefer unit ground goals.

**PreferGround:** Prefer clauses without variables.

**PreferNonGround:** Prefer clauses with variables.

**PreferProcessed:** Prefer clauses that have already been processed once and have been eliminated from the set of processed clauses due to interreduction (forward contraction).

**PreferNew:** Prefer new clauses, i.e. clauses that are processed for the first time.

**PreferGoals:** Prefer goals (all negative clauses).

**PreferNonGoals:** Prefer non goals, i.e. facts with at least one positive literal.

**PreferUnits:** Prefer unit clauses (clauses with one literal).

**PreferNonUnits:** Prefer non-unit clauses.

**PreferHorn:** Prefer Horn clauses (clauses with no more than one positive literals).

**PreferNonHorn:** Prefer non-Horn clauses.

**ConstPrio:** Assign the same priority to all clauses.

**ByLiteralNumber:** Give a priority according to the number of literals, i.e. always prefer a clause with fewer literals to one with more literals.

**ByDerivationDepth:** Prefer clauses which have a short derivation depth, i.e. give a priority based on the length of the longest path from the clause to an axiom in the derivation tree. Counts generating inferences only.

**ByDerivationSize:** Prefer clauses which have been derived with a small number of (generating) inferences.

**ByNegLitDist:** Prefer goals to non-goals. Among goals, prefer goals with fewer literals and goals with ground literals (more exactly: the priority is increased by 1 for a ground literal and by 3 for a non-ground literal. Clauses with lower values are selected before clauses with higher values).

**ByGoalDifficulty:** Prefer goals to non-goals. Select goals based on a simple estimate of their difficulty: First unit ground goals, then unit goals, then ground goals, then other goals.

**SimulateSOS:** Use the priority system to simulate Set-Of-Support. This prefers all initial clauses and all Set-Of-Support clauses. Some non-SOS-clauses will be generated, but not selected for processing. This is neither well tested nor a particularly good fit with E's calculus, but can be used as one among many heuristics. If you try a pure SOS strategy, you also should set `--restrict-literal-comparisons` and run the prover without literal selection enabled.

**PreferHorn:** Prefer Horn clauses (note: includes units).

**PreferNonHorn:** Prefer non-Horn clauses.

**PreferUnitAndNonEq:** Prefer all unit clauses and all clauses without equational literal. This was an attempt to model some restricted calculi used e.g. in Gandalf [Tam97], but did not quite work out.

**DeferNonUnitMaxEq:** Prefer everything except for non-unit clauses with a maximal equational literal ("Don't paramodulate if its too expensive"). See above, same result.

**ByCreationDate:** Return the creation date of the clause as priority. This imposes a FIFO equivalence class on clauses. Clauses generated from the same given clause are grouped together (and can be ordered with any evaluation function among each other).

**PreferWatchlist** Prefer clauses on the watchlist (see 4.4).

**DeferWatchlist** Defer clauses on the watchlist (see above).

Please note that careless use of certain priority functions can make the prover incomplete for the general case.

#### 4.1.2 Generic Weight Functions

Generic weight functions are templates for functions taking a clause and returning a weight (i.e. an estimate of the usefulness) for it, where a lower weight means that the corresponding clause should be processed before a clause with a higher weight. A generic weight function is combined with a priority function and instantiated with a set of parameters to yield a clause evaluation function.

You can specify an instantiated generic weight function as described in this rule<sup>8</sup>:

```
<weight-fun> ::= Clauseweight '(' <prio-fun> ', <int>, <int>,
                        <float> ')' ||
                Refinedweight '(' <prio-fun> ', <int>, <int>,
                        <float>, <float>, <float> ')' ||
                Orientweight '(' <prio-fun>, <int>, <int>,
                        <float>, <float>, <float> ')' ||
                Simweight '(' <prio-fun>, <float>, <float>,
                        <float>, <float> ')' ||
                FIFOweight '(' <prio-fun> ')' ||
                LIFOweight '(' <prio-fun> ')'
```

**Clauseweight(prio, fweight, vweight, pos\_mult)**: This is the basic symbol counting heuristic. Variables are counted with weight **vweight**, function symbols with weight **fweight**. The weight of positive literals is multiplied by **pos\_mult** before being added into the final weight.

**Refinedweight(prio, fweight, vweight, term\_pen, lit\_pen, pos\_mult)**: This weight function is very similar to the first one. It differs only in that it takes the effect of the term ordering into account. In particular, the weight of a term that is maximal in its literal is multiplied by **term\_pen**, and the weight of maximal literals is multiplied by **lit\_pen**.

**Orientweight(prio, fweight, vweight, term\_pen, lit\_pen, pos\_mult)**: This weight function is a slight variation of **Refinedweight()**. In this case, the weight of *both* terms of an unorientable literal is multiplied by a penalty **term\_pen**.

**Simweight(prio, equal\_weight, vv\_clash, vt\_clash, tt\_clash)**: This weight function is intended to return a low weight for literals in which the two terms are very similar. It does not currently work very well even for unit clauses – RTFS (in `<che_simweight.c>`) to find out more.

---

<sup>8</sup>Note that there now are many additional generic weight functions not yet documented.

**FIFOWeight(prio):** This weight function assigns weights that increase in a strictly monotonic manner, i.e. it realizes a *first-in/first-out* strategy if used all by itself. This is the most obviously fair strategy.

**LIFOWeight(prio):** This weight function assigns weights that decrease in a strictly monotonic manner, i.e. it realizes a *last-in/first-out* strategy if used all by itself (which, of course, would be unfair and result in an extremely incomplete prover).

### 4.1.3 Clause Evaluation Functions

A clause evaluation function is constructed by instantiating a generic weight function. It can either be specified directly, or specified and given a name for later reference at once:

```
<eval-fun>          ::= <ident>          ||
                      <weight-fun>       ||
                      <eval-fun-def>
<eval-fun-def>      ::= <ident> = <weight-fun>
<eval-fun-def-list> ::= <eval-fun-def>*
```

Of course a single identifier is only a valid evaluation function if it has been previously defined in a `<eval-fun-def>`. It is possible to define the value of an identifier more than once, in which case later definitions take precedence to former ones.

Clause evaluation functions can be defined on the command line with the `-D` (`--define-weight-function`) option, followed by a `<eval-fun-def-list>`.

*Example:*

```
eprover -D"ex1=Clauseweight(ConstPrio,2,1,1) \
          ex2=FIFOWeight(PreferGoals)" ...
```

sets up the prover to know about two evaluation function `ex1` and `ex2` (which supposedly will be used later on the command line to define one or more heuristics). The double quotes are necessary because the brackets and the commas are special characters for most shells

There are a variety of clause evaluation functions predefined in the variable `DefaultWeightFunctions`, which can be found in `che_proofcontrol.c`. See also sections 4.4 and 4.5, which cover some of the more complex weight functions of E.

#### 4.1.4 Heuristics

A heuristic defines how many selections are to be made according to one of several clause evaluation functions. Syntactically,

```
<heu-element>    ::= <int> '*' <eval-fun>
<heuristic>      ::= '(' <heu-element> (,<heu-element>)* ')' ||
                  <ident>
<heuristic-def> ::= <ident> = <heuristic> ||
                  <heuristic>
```

As above, a single identifier is only a valid heuristic if it has been defined in `<heuristic-def>` previously. A `<heuristic-def>` which degenerates to a simple heuristic defines a heuristic with name `Default` (which the prover will automatically choose if no other heuristic is selected with the `-x` (`--expert-heuristic`)).

*Example:* To continue the above example,

```
eprover -D"ex1=Clauseweight(ConstPrio,2,1,1) \
        ex2=FIFOWeight(PreferGoals)"
        -H"new=(3*ex1,1*ex2)" \
        -x new LUSK3.lop
```

will run the prover on a problem file named `LUSK3.lop` with a heuristic that chooses 3 out of every 4 clauses according to a simple symbol counting heuristic and the last clause first among goals and then among other clauses, selecting by order of creation in each of these two classes.

## 4.2 Term Orderings

E currently supports two families of orderings: The *Knuth-Bendix-Ordering* (KBO), which is used by default, and the *Lexicographical Path Ordering* (LPO). The KBO is weight-based and uses a precedence on function symbols to break ties. Consequently, to specify a concrete KBO, we need a weight function that assigns a weight to all function symbols, and a precedence on those symbols.

The LPO is based on a lexicographic comparison of symbols and subterms, and is fully specified by giving just a precedence.

Currently it is possible to explicitly specify an arbitrary (including incomplete or empty) precedence, or to use one of several precedence generating schemes. Similarly, there is a number of predefined weight function and the ability to assign arbitrary weights to function and predicate symbols.

The simplest way to get a reasonable term ordering is to specify *automatic* ordering selection using the `-tAuto` option.

Options controlling the choice of term ordering:

`-term-ordering=<arg>`  
`-t<arg>` Select a term ordering class (or automatic selection). Supported arguments are at least LPO, LP04 (for a much faster new implementation of LPO), KBO, and Auto. If Auto is selected, all aspects of the term ordering are fixed, additional options will be (or at least should be) silently ignored.  
`--order-precedence-generation=<arg>`  
`-G <arg>` Select a precedence generation scheme (see below).  
`--order-weight-generation=<arg>`  
`-w <arg>` Select a symbol weight function (see below).  
`--order-constant-weight=<arg>`  
`-c <arg>` Modify any symbol weight function by assigning a special weight to constant function symbols.  
`--precedence[=<arg>]`  
Describe a (partial) precedence for the term ordering. The argument is a comma-separated list of precedence chains, where a precedence chain is a list of function symbols (which all have to appear in the proof problem), connected by `>`, `<`, or `=` (to denote equivalent symbols). If this option is used in connection with `--order-precedence-generation`, the partial ordering will be completed using the selected method, otherwise the prover runs with a non-ground-total ordering. The option without the optional argument is equivalent to `--precedence=` (the empty precedence). There is a drawback to using `--precedence`: Normally, total precedences are represented by mapping symbols to a totally ordered set (small integers) which can be compared using standard machine instructions. The used data structure is linear in the number  $n$  of function symbols. However, if `--precedence` is used, the prover allocates (and completes) a  $n \times n$  lookup table to efficiently represent an arbitrary partial ordering. If  $n$  is very big, this matrix takes up significant space, and takes a long time to compute in the first place. This is unlikely to be a problem unless there are at least hundreds of symbols.

**--order-weights=<arg>**

Give explicit weights to function symbols. The argument syntax is a comma-separated list of items of the form **f:w**, where **f** is a symbol from the specification, and **w** is a non-negative integer. Note that at best very simple checks are performed, so you can specify weights that do not obey the KBO weight constraints. Behaviour in this case is undefined. If all your weights are positive, this is unlikely to happen.

Since KBO needs a total weight function, E always uses a weight generation scheme in addition to the user-defined options. You may want to use **-wconstant** for predictable behaviour.

**--lpo-recursion-limit[=<arg>]**

Limits the recursion depth of LPO comparison. This is useful in rare cases where very large term comparisons can lead to stack overflow issues. It does not change completeness, but may lead to unnecessary inferences in rare cases (Note: By default, recursion depth is limited to 1000. To get effectively unlimited recursion depth, use this option with an outrageously large argument. Don't forget to increase process stack size with **limit/ulimit** from your favourite shell).

#### 4.2.1 Precedence Generation Schemes

Precedence generation schemes are based on syntactic features of the symbol and the input clause set, like symbol arity or number of occurrences in the formula. At least the following options are supported as argument to **--order-precedence-generation**:

**unary\_first**: Sort symbols by arity, with the exception that unary symbols come first. Frequency is used as a tie breaker (rarer symbols are greater).

**unary\_freq**: Sort symbols by frequency (rarer symbols are bigger), with the exception that unary symbols come first. Yes, this should better be named **unary\_invfreq** for consistency, but is not...

**arity**: Sort symbols by arity (symbols with higher arity are larger).

**invarity**: Sort symbols by arity (symbols with higher arity are smaller).

**const\_max**: Sort symbols by arity (symbols with higher arity are larger), but make constants the largest symbols. This is allegedly used by SPASS [Wei01] in some configurations.

**const\_min**: Sort symbols by arity (symbols with higher arity are smaller), but make constants the smallest symbols. Provided for reasons of symmetry.

**freq**: Sort symbols by frequency (frequently occurring symbols are larger). Arity is used as a tie breaker.



**invfreq:** Sort symbols by frequency (frequently occurring symbols are smaller).  
In our experience, this is one of the best general-purpose precedence generation schemes.

**invfreqconstmin:** Same as **invfreq**, but make constants always smaller than everything else.

**invfreqhack:** As **invfreqconstmin**, but unary symbols with maximal frequency become largest.

#### 4.2.2 Weight Generation Schemes

Weight generation schemes are based on syntactic features of the symbol and the input clause set, or on the predefined *precedence*. The following options are available for `--order-weight-generation`.

**firstmaximal0:** Give the same arbitrary (positive) weight to all function symbols except to the first maximal one encountered (order is arbitrary), which is given weight 0.

**arity:** Weight of a function symbol  $f|_n$  is  $n + 1$ , i.e. its arity plus one.

**aritymax0:** As **arity**, except that the first maximal symbol is given weight 0.

**modarity:** Weight of a function symbol  $f|_n$  is  $n+c$ , where  $c$  is a positive constant (`W_TO_BASEWEIGHT`, which has been 4 since the dawn of time).

**modaritymax0:** As **modarity**, except that the first maximal symbol is given weight 0.

**aritysquared:** Weight of a symbol  $f|_n$  is  $n^2 + 1$ .

**aritysquaredmax0:** As **aritysquared**, except that the first maximal symbol is given weight 0.

**invarity:** Let  $m$  be the largest arity of any symbol in the signature. Weight of a symbol  $f|_n$  is  $m - n + 1$ .

**invaritymax0:** As **invarity**, except that the first maximal symbol is given weight 0.

**invaritysquared:** Let  $m$  be the largest arity of any symbol in the signature. Weight of a symbol  $f|_n$  is  $m^2 - n^2 + 1$ .

**invaritysquaredmax0:** As **invaritysquared**, except that the first maximal symbol is given weight 0.

**precedence:** Let  $<$  be the (pre-determined) precedence on function symbols  $F$  in the problem. Then the weight of  $f$  is given by  $|g|g < f| + 1$  (the number of symbols smaller than  $f$  in the precedence increased by one).

**invprecedence:** Let  $<$  be the (pre-determined) precedence on function symbols  $F$  in the problem. Then the weight of  $f$  is given by  $|g|f < g| + 1$  (the number of symbols larger than  $f$  in the precedence increased by one).

**freqcount:** Make the weight of a symbol the number of occurrences of that symbol in the (potentially preprocessed) input problem.

**invfreqcount:** Let  $m$  be the number of occurrences of the most frequent symbol in the input problem. The weight of  $f$  is  $m$  minus the number of occurrences of  $f$  in the input problem.

**freqrank:** Sort all function symbols by frequency of occurrence (which induces a total quasi-ordering). The weight of a symbol is the rank of its equivalence class, with less frequent symbols getting lower weights.

**invfreqrank:** Sort all function symbols by frequency of occurrence (which induces a total quasi-ordering). The weight of a symbol is the rank of its equivalence class, with less frequent symbols getting higher weights.

**freqranksquare:** As **freqrank**, but weight is the square of the rank.

**invfreqranksquare:** As **invfreqrank**, but weight is the square of the rank.

**invmodfreqrank:** Sort all function symbols by frequency of occurrence (which induces a total quasi-ordering). The weight of an equivalence class is the sum of the cardinality of all smaller classes (+1). The weight of a symbol is the weight of its equivalence classes. Less frequent symbols get higher weights.

**invmodfreqrankmax0:** As **invmodfreqrank**, except that the first maximal symbol is given weight 0.

**constant:** Give the same arbitrary positive weight to all function symbols.

### 4.3 Literal Selection Strategies

The superposition calculus allows the *selection* of arbitrary negative literals in a clause and only requires generating inferences to be performed on these literals. E supports this feature and implements it via manipulations of the literal ordering. Additionally, E implements strategies that allow inferences into maximal positive literals and selected negative literals. A selection strategy is selected with the option `--literal-selection-strategy`. Currently, at least the following strategies are implemented:

**NoSelection:** Perform ordinary superposition without selection.

**NoGeneration:** Do not perform any generating inferences. This strategy is not complete, but applying it to a formula generates a normal form that does not contain any tautologies or redundant clauses.

**SelectNegativeLiterals:** Select all negative literals. For Horn clauses, this implements the maximal literal positive unit strategy [Der91] previously realized separately in E.

**SelectPureVarNegLiterals:** Select the first negative literal of the form  $X \simeq Y$ .

**SelectLargestNegLit:** Select the largest negative literal (by symbol counting, function symbols count as 2, variables as 1).

**SelectSmallestNegLit:** As above, but select the smallest literal.

**SelectDiffNegLit:** Select the negative literal in which both terms have the largest size difference.

**SelectGroundNegLit:** Select the first negative ground literal for which the size difference between both terms is maximal.

**SelectOptimalLit:** If there is a ground negative literal, select as in the case of **SelectGroundNegLit**, otherwise as in **SelectDiffNegLit**.

Each of the strategies that do actually select negative literals has a corresponding counterpart starting with P that additionally allows paramodulation into maximal positive literals<sup>9</sup>.

*Example:* Some problems become a lot simpler with the correct strategy. Try e.g.

```
eprover --literal-selection-strategy=NoSelection \
        GRP001-1+rm_eq_rstfp.lop
eprover --literal-selection-strategy=SelectLargestNegLit \
        GRP001-1+rm_eq_rstfp.lop
```

You will find the file `GRP001-1+rm_eq_rstfp.lop` in the `E/PROVER` directory.

As we aim at replacing the vast number of individual literal selection functions with a more abstract mechanism, we refrain from describing all of the currently implemented functions in detail. If you need information about the set of implemented functions, run `eprover -W none`. The individual functions are implemented and somewhat described in `E/HEURISTICS/che_litselection.h`.

---

<sup>9</sup>Except for **SelectOptimalLit**, where the resulting strategy, **PSelectOptimalLit** will allow paramodulation into positive literals only if no ground literal has been selected.

## 4.4 The Watchlist Feature

Since public release 0.81, E supports a *watchlist*. A watchlist is a user-defined set of clauses. Whenever the prover encounters<sup>10</sup> a clause that subsumes one or more clauses from the watchlist, those clauses are removed from it. The saturation process terminates if the watchlist is empty (or, of course, if a saturated state or the empty clause have been reached).

There are two uses for a watchlist: To guide the proof search (using a heuristic that prefers clauses on the watchlist), or to find purely constructive proofs for clauses on the watchlist.

If you want to guide the proof search, place clauses you believe to be important lemmata onto the watchlist. Also include the empty clause to make sure that the prover will not terminate prematurely. You can then use a clause selection heuristic that will give special consideration to clauses on the watchlist. This is currently supported via the *priority functions* `PreferWatchlist` and `DeferWatchlist`. A clause evaluation function using `PreferWatchlist` will always select clauses which subsume watchlist clauses first. Similarly, using `DeferWatchlist` can be used to put the processing of watchlist clauses off.

There is a predefined clause selection heuristic `UseWatchlist` (select it with `-xUseWatchlist`) that will make sure that watchlist clauses are selected relatively early. It is a strong general purpose heuristic, and will maintain completeness of the prover. This should allow easy access to the watchlist feature even if you don't yet feel comfortable with specifying your own heuristics.

To generate constructive proofs of clauses, just place them on the watch list and select output level 4 or greater (see section 6.3). Steps effecting the watch list will be marked in the PCL2 output file. If you use the *eproof* script for proof output or run *epclextract* of your own, subproof for watchlist steps will be automatically extracted.

Note that this forward reasoning is not complete, i.e. the prover may never generate a given watchlist clause, even if it would be trivial to prove it via refutation.

Options controlling the use of the watch list:

- |  |   |
|--|---|
| <code>--watchlist=&lt;arg&gt;</code>       | Select a file containing the watch list clauses. Syntax should be the same syntax as your proof problem (E-LOP, TPTP or TSTP). Just write down a list of clauses. |
| <code>--no-watchlist-simplification</code> | By default, watch list clauses are simplified with respect to the current set P. Use this option to disable the feature.  |

---

<sup>10</sup>Clauses are checked against the watchlist after normalization, both when they are inserted into U or if they are selected for processing.

## 4.5 Learning Clause Evaluation Functions

E can use a knowledge base generated by analyzing many successful proof attempts to guide its search, i.e. it can *learn* what kinds of clauses are likely to be useful for a proof and which ones are likely superfluous. The details of the learning mechanism can be found in [Sch00, Sch01]. Essentially, an inference protocol is analyzed, useful and useless clauses are identified and generalized into *clause patterns*, and the resulting information is stored in a knowledge base. Later, new clauses that match a pattern are evaluated accordingly.

### 4.5.1 Creating Knowledge Bases

An E knowledge base is a directory containing a number of files, storing both the knowledge and configuration information. Knowledge bases are generated with the tool `ekb.create`. If no argument is given, `ekb.create` will create a knowledge base called `E.KNOWLEDGE` in the current directory.

You can run `ekb.create -h` for more information about the configuration. However, the defaults are usually quite sufficient.

### 4.5.2 Populating Knowledge Bases

The knowledge base contains information gained from clausal PCL2 protocols of E. In a first step, information from the protocol is abstracted into a more compact form. A number of clauses is selected as training examples, and annotations about their role are computed. The result is a list of annotated clauses and a list of the axioms (initial clauses) of the problem. This step can be performed using the program `direct.examples`<sup>11</sup>.

In a second step, the collected information is integrated into the knowledge base. For this purpose, the program `ekb.insert` can be used. However, it is probably more convenient to use the single program `ekb.ginsert`, which directly extracts all pertinent information from a PCL2 protocol and inserts it into a designated knowledge base.

The program `ekb.delete` will delete an example from a knowledge base. This process is not particularly efficient, as the whole knowledge base is first parsed.

### 4.5.3 Using Learned Knowledge

The knowledge in a knowledge base can be utilized by the two clause evaluation functions `TSMWeight()` and `TSMRWeight()`. Both compute a modification weight based on the learned knowledge, and apply it to a conventional symbol-counting base weight (similar to `Clauseweight()` for `TSMWeight()` and `Refinedweight()` for `TSMRWeight()`). An example command line is:

```
eprover -x'(1*TSMWeight(ConstPrio, 1, 1, 2, flat, E.KNOWLEDGE,
100000,1.0,1.0,Flat,IndexIdentity,100000,-20,20,-2,-1,0,2))'
```

---

<sup>11</sup>The name is an historical accident and has no significance anymore

There are also two fully predefined learning clause selection heuristics. Select them with `-xUseTSM1` (for some influence of the learned knowledge) or `-xUseTSM2` (for a lot of influence of the learned knowledge).

## 4.6 Other Options

# 5 Input Language

## 5.1 LOP

E natively uses E-LOP, a dialect of the LOP language designed for SETHEO. At the moment, your best bet is to retrieve the LOP description from the E web site [Sch99] and/or check out the examples available from it. LOP is very close to Prolog, and E can usually read many fully declarative Prolog files if they do not use arithmetic or rely on predefined symbols. Plain SETHEO files usually also work very well. There are a couple of minor differences, however:

- `equal()` is an interpreted symbol for E. It normally does not carry any meaning for SETHEO (unless equality axioms are added).
- SETHEO allows the same identifier to be used as a constant, a non-constant function symbol and a predicate symbol. E encodes all of these as ordinary function symbols, and hence will complain if a symbol is used inconsistently.
- E allows the use of both `=` and `=>` as infix symbols for equality. `a=b` is equivalent to `equal(a,b)` for E.
- E does not support constraints or SETHEO build-in symbols. This should not usually affect pure theorem proving tasks.
- E normally treats procedural clauses exactly as it treats declarative clauses. Query clauses (clauses with an empty head and starting with `?-`, e.g. `?-~p(X), q(X).` can optionally be used to define the a set of *goal clauses* (by default, all negative clauses are considered to be goals). At the moment, this information is only used for the initial set of support (with `--sos-uses-input-types`). Note that you can still specify arbitrary clauses as query clauses, since LOP supports negated literals.

## 5.2 TPTP Format

The TPTP [Sut05] is a library of problems for automated theorem prover. Problems in the TPTP are written in TPTP syntax. There are two major versions of the TPTP syntax, both of which are supported by E.

Version 2<sup>12</sup> of the TPTP syntax was used up for TPTP releases previous to TPTP 3.0.0.

---

<sup>12</sup>Version 1 allowed the specification of problems in clause normal form only. Version 2 is a conservative extension of version 1 and adds support for full first order formulas.

The current version 3 of the TPTP syntax, described in [SSCG06], covers both input problems and both proof and model output using one consistent formalism. It has been used as the native format for TPTP releases since TPTP 3.0.0.

Parsing in TPTP format version 2 is enabled by the options `--tptp-in`, `tptp2-in`, `--tptp-format` and `--tptp2-format`. The last two options also select TPTP 2 format for the output of normal clauses during and after saturation. Proof output will be in PCL2 format, however.

As an alternative, E also supports TPTP-3 syntax with the options `--tstp-in`, `tstp3-in`, `--tstp-format` and `--tstp3-format`. The last two options will also enable TPTP-3 format for proof output. Note that many of E's support tools still require PCL2 format. Various tools for processing TPTP-3 proof format are available via the TPTP web-site, <http://www.tptp.org>.

In either TPTP format, clauses and formulas with TPTP type `conjecture` or *negated-conjecture* (TPTP-3 only) are considered goal clauses for the `--sos-uses-input-types` option.

## 6 Output...or how to interpret what you see

E has several different output levels, controlled by the option `-l` or `--output-level`. Level 0 prints nearly no output except for the result. Level 1 is intended to give humans a somewhat readable impression of what is going on inside the inference engine. Levels 3 to 6 output increasingly more information about the inside processes in PCL2 format. At level 4 and above, a (large) superset of the proof inferences is printed. You can use the `epclextract` utility in `E/PROVER/` to extract a simple proof object.

In Level 0 and 1, everything E prints is either a clause that is implied by the original axioms, or a comment (or, very often, both).

### 6.1 The Bare Essentials

In silent mode (`--output-level=0`, `-s` or `--silent`), E will not print any output during saturation. It will print a one-line comment documenting the state of the proof search after termination. The following possibilities exist:

- The prover found a proof. This is denoted by the output string

```
# Proof found!
```

- The problem does not have a proof, i.e. the specification is satisfiable (and E can detect this):

```
# No proof found!
```

Ensuring the completeness of a prover is much harder than ensuring correctness. Moreover, proofs can easily be checked by analyzing the output

of the prover, while such a check for the absence of proofs is rarely possible. I do believe that the current version of E is both correct and complete<sup>13</sup> but my belief in the former is stronger than my belief in the later. ....

- A (hard) resource limit was hit. For memory this can be either due to a per process limit (set with `limit` or the prover option `--memory-limit`), or due to running out of virtual memory. For CPU time, this case is triggered if the per process CPU time limit is reached and signaled to the prover via a `SIGXCPU` signal. This limit can be set with `limit` or, more reliable, with the option `--cpu-limit`. The output string is one of the following two, depending on the exact reason for termination:

```
# Failure: Resource limit exceeded (memory)
# Failure: Resource limit exceeded (time)
```

- A user-defined limit was reached during saturation, and the saturation process was stopped gracefully. Limits include number of processed clauses, number of total clauses, and cpu time (as set with `--soft-cpu-limit`). The output string is

```
# Faiure: User resource limit exceeded!
```

...and the user is expected to know which limit he selected.

- Normally, E is complete. However, if the option `--delete-bad-limit` is given or if automatic mode in connection with a memory limit is used, E will periodically delete clauses it deems unlikely to be processed to avoid running out of memory. In this case, completeness cannot be ensured any more. This effect manifests itself extremely rarely. If it does, E will print the following string:

```
# Failure: Out of unprocessed clauses!
```

This is roughly equivalent to Otter's `SOS empty` message.

- Finally, it is possible to chose restricted calculi when starting E. This is useful if E is used as a normalization tool or as a preprocessor or lemma generator. In this case, E will print a corresponding message:

```
# Clause set closed under restricted calculus!
```

---

<sup>13</sup>Unless the prover runs out of memory (see below), the user selects an unfair strategy (in which case the prover may never terminate), or some strange and unexpected things happen.



## 6.2 Impressing your Friends

If you run E without selection an output level (or by setting it explicitly to 1), E will print each non-tautological, non-subsumed clause it processes as a comment. It will also print a hash ('#') for each clause it tries to process but can prove to be superfluous.

This mode gives some indication of progress, and as the output is fairly restricted, does not slow the prover down too much.

For any output level greater than 0, E will also print statistical information about the proof search and final clause sets. The data should be fairly self-explaining.

## 6.3 Detailed Reporting

At output levels greater than 1, E prints certain inferences in PCL2 format<sup>14</sup>. At level 2, it only prints generating inferences. At level 4, it prints all generating and modifying inferences, and at level 6 it also prints PCL steps giving a lot of insight into the internal operation of the inference engine. This protocol is fairly readable and, from level 4 on can be used to check the proof with the utility `checkproof` provided with the distribution.

## 6.4 Requesting Specific Results

There are two additional kinds of information E can provide beyond the normal output during proof search: Statistical information and final clause sets (with additional information).

First, E can give you some technical information about the conditions it runs under.

The option `--print-pid` will make E printing its process id as a comment, in the format `# Pid: XXX`, where XXX is an integer number. This is useful if you want to send signals to the prover (in particular, if you want to terminate the prover) to control it from the outside.

The option `-R (--resources-info)` will make E print a summary of used system resources after graceful termination:

```
# User time           : 0.010 s
# System time         : 0.020 s
# Total time          : 0.030 s
# Maximum resident set size: 0 pages
```

Most operating systems do not provide a valid value for the resident set size and other memory-related resources, so you should probably not depend on the last value to carry any meaningful information. The time information is required by most standards and should be useful for all tested operating systems.

---

<sup>14</sup>PCL2 is a proof output protocol language currently being designed by me as a successor to PCL [DS94a, DS94b, DS96].

E can be used not only as a prover, but as a normalizer for formulae or as a lemma generator. In this cases, you will not only want to know if E found a proof, but also need some or all of the derived clauses, possibly with statistical information for filtering. This is supported with the `--print-saturated` and `--print-sat-info` options for E.

The option `--print-saturated` takes as its argument a string of letters, each of which represents a part of the total set of clauses E knows about. The following table contains the meaning of the individual letters:

- e** Processed positive unit clauses (*Equations*).
- i** Processed negative unit clauses (*Inequations*).
- g** Processed non-unit clauses (except for the empty clause, which, if present, is printed separately). The above three sets are interreduced and all selected inferences between them have been computed.
- E** Unprocessed positive unit clauses.
- I** Unprocessed negative unit clauses.
- G** Unprocessed non-unit clause (this set may contain the empty clause in very rare cases).
- a** Print equality axioms (if equality is present in the problem). This letter prints axioms for reflexivity, symmetry, and transitivity, and a set of substitutivity axioms, one for each argument position of every function symbol and predicate symbol.
- A** As **a**, but print a single substitutivity axiom covering all positions for each symbol.

The short form, `-S`, is equivalent to `--print-saturated=eigEIG`. If the option `--print-sat-info` is set, then each of the clauses is followed by a comment of the form `# info(id, pd, pl, sc, cd, nl, no, nv)`. The following table explains the meaning of these values:

- id** Clause ident (probably only useful internally)
- pd** Depth of the derivation graph for this clause
- pl** Number of nodes in the derivation grap
- sc** Symbol count (function symbols and variables)
- cd** Depth of the deepest term in the clause
- nl** Number of literals in the clause
- no** Number of variable occurences
- nv** Number of different variables

## A License

The standard distribution of E is free software. You can use, modify and copy it under the terms of the GNU General Public License. You may also have bought a commercial version of E from Safelogic A.B. in Gothenburg, Sweden. In this case, you are bound by whatever license you agreed to. If you are in doubt about which version of E you have, run `eprover -V` or `eprover -h`.

### GNU GENERAL PUBLIC LICENSE Version 2, June 1991

Copyright (C) 1989, 1991 Free Software Foundation, Inc.  
59 Temple Place, Suite 330, Boston, MA 02111-1307 USA  
Everyone is permitted to copy and distribute verbatim copies  
of this license document, but changing it is not allowed.

#### Preamble

The licenses for most software are designed to take away your freedom to share and change it. By contrast, the GNU General Public License is intended to guarantee your freedom to share and change free software--to make sure the software is free for all its users. This General Public License applies to most of the Free Software Foundation's software and to any other program whose authors commit to using it. (Some other Free Software Foundation software is covered by the GNU Library General Public License instead.) You can apply it to your programs, too.

When we speak of free software, we are referring to freedom, not price. Our General Public Licenses are designed to make sure that you have the freedom to distribute copies of free software (and charge for this service if you wish), that you receive source code or can get it if you want it, that you can change the software or use pieces of it in new free programs; and that you know you can do these things.

To protect your rights, we need to make restrictions that forbid anyone to deny you these rights or to ask you to surrender the rights. These restrictions translate to certain responsibilities for you if you distribute copies of the software, or if you modify it.

For example, if you distribute copies of such a program, whether gratis or for a fee, you must give the recipients all the rights that you have. You must make sure that they, too, receive or can get the source code. And you must show them these terms so they know their rights.

We protect your rights with two steps: (1) copyright the software, and (2) offer you this license which gives you legal permission to copy, distribute and/or modify the software.

Also, for each author's protection and ours, we want to make certain that everyone understands that there is no warranty for this free software. If the software is modified by someone else and passed on, we want its recipients to know that what they have is not the original, so that any problems introduced by others will not reflect on the original authors' reputations.

Finally, any free program is threatened constantly by software patents. We wish to avoid the danger that redistributors of a free program will individually obtain patent licenses, in effect making the program proprietary. To prevent this, we have made it clear that any patent must be licensed for everyone's free use or not licensed at all.

The precise terms and conditions for copying, distribution and modification follow.

#### GNU GENERAL PUBLIC LICENSE TERMS AND CONDITIONS FOR COPYING, DISTRIBUTION AND MODIFICATION

0. This License applies to any program or other work which contains a notice placed by the copyright holder saying it may be distributed under the terms of this General Public License. The "Program", below, refers to any such program or work, and a "work based on the Program" means either the Program or any derivative work under copyright law: that is to say, a work containing the Program or a portion of it, either verbatim or with modifications and/or translated into another language. (Hereinafter, translation is included without limitation in the term "modification".) Each licensee is addressed as "you".

Activities other than copying, distribution and modification are not covered by this License; they are outside its scope. The act of running the Program is not restricted, and the output from the Program is covered only if its contents constitute a work based on the Program (independent of having been made by running the Program). Whether that is true depends on what the Program does.

1. You may copy and distribute verbatim copies of the Program's source code as you receive it, in any medium, provided that you conspicuously and appropriately publish on each copy an appropriate copyright notice and disclaimer of warranty; keep intact all the notices that refer to this License and to the absence of any warranty; and give any other recipients of the Program a copy of this License along with the Program.

You may charge a fee for the physical act of transferring a copy, and you may at your option offer warranty protection in exchange for a fee.

2. You may modify your copy or copies of the Program or any portion of it, thus forming a work based on the Program, and copy and

distribute such modifications or work under the terms of Section 1 above, provided that you also meet all of these conditions:

- a) You must cause the modified files to carry prominent notices stating that you changed the files and the date of any change.
- b) You must cause any work that you distribute or publish, that in whole or in part contains or is derived from the Program or any part thereof, to be licensed as a whole at no charge to all third parties under the terms of this License.
- c) If the modified program normally reads commands interactively when run, you must cause it, when started running for such interactive use in the most ordinary way, to print or display an announcement including an appropriate copyright notice and a notice that there is no warranty (or else, saying that you provide a warranty) and that users may redistribute the program under these conditions, and telling the user how to view a copy of this License. (Exception: if the Program itself is interactive but does not normally print such an announcement, your work based on the Program is not required to print an announcement.)

These requirements apply to the modified work as a whole. If identifiable sections of that work are not derived from the Program, and can be reasonably considered independent and separate works in themselves, then this License, and its terms, do not apply to those sections when you distribute them as separate works. But when you distribute the same sections as part of a whole which is a work based on the Program, the distribution of the whole must be on the terms of this License, whose permissions for other licensees extend to the entire whole, and thus to each and every part regardless of who wrote it.

Thus, it is not the intent of this section to claim rights or contest your rights to work written entirely by you; rather, the intent is to exercise the right to control the distribution of derivative or collective works based on the Program.

In addition, mere aggregation of another work not based on the Program with the Program (or with a work based on the Program) on a volume of a storage or distribution medium does not bring the other work under the scope of this License.

3. You may copy and distribute the Program (or a work based on it, under Section 2) in object code or executable form under the terms of Sections 1 and 2 above provided that you also do one of the following:

- a) Accompany it with the complete corresponding machine-readable source code, which must be distributed under the terms of Sections 1 and 2 above on a medium customarily used for software interchange; or,

b) Accompany it with a written offer, valid for at least three years, to give any third party, for a charge no more than your cost of physically performing source distribution, a complete machine-readable copy of the corresponding source code, to be distributed under the terms of Sections 1 and 2 above on a medium customarily used for software interchange; or,

c) Accompany it with the information you received as to the offer to distribute corresponding source code. (This alternative is allowed only for noncommercial distribution and only if you received the program in object code or executable form with such an offer, in accord with Subsection b above.)

The source code for a work means the preferred form of the work for making modifications to it. For an executable work, complete source code means all the source code for all modules it contains, plus any associated interface definition files, plus the scripts used to control compilation and installation of the executable. However, as a special exception, the source code distributed need not include anything that is normally distributed (in either source or binary form) with the major components (compiler, kernel, and so on) of the operating system on which the executable runs, unless that component itself accompanies the executable.

If distribution of executable or object code is made by offering access to copy from a designated place, then offering equivalent access to copy the source code from the same place counts as distribution of the source code, even though third parties are not compelled to copy the source along with the object code.

4. You may not copy, modify, sublicense, or distribute the Program except as expressly provided under this License. Any attempt otherwise to copy, modify, sublicense or distribute the Program is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

5. You are not required to accept this License, since you have not signed it. However, nothing else grants you permission to modify or distribute the Program or its derivative works. These actions are prohibited by law if you do not accept this License. Therefore, by modifying or distributing the Program (or any work based on the Program), you indicate your acceptance of this License to do so, and all its terms and conditions for copying, distributing or modifying the Program or works based on it.

6. Each time you redistribute the Program (or any work based on the Program), the recipient automatically receives a license from the original licensor to copy, distribute or modify the Program subject to

these terms and conditions. You may not impose any further restrictions on the recipients' exercise of the rights granted herein. You are not responsible for enforcing compliance by third parties to this License.

7. If, as a consequence of a court judgment or allegation of patent infringement or for any other reason (not limited to patent issues), conditions are imposed on you (whether by court order, agreement or otherwise) that contradict the conditions of this License, they do not excuse you from the conditions of this License. If you cannot distribute so as to satisfy simultaneously your obligations under this License and any other pertinent obligations, then as a consequence you may not distribute the Program at all. For example, if a patent license would not permit royalty-free redistribution of the Program by all those who receive copies directly or indirectly through you, then the only way you could satisfy both it and this License would be to refrain entirely from distribution of the Program.

If any portion of this section is held invalid or unenforceable under any particular circumstance, the balance of the section is intended to apply and the section as a whole is intended to apply in other circumstances.

It is not the purpose of this section to induce you to infringe any patents or other property right claims or to contest validity of any such claims; this section has the sole purpose of protecting the integrity of the free software distribution system, which is implemented by public license practices. Many people have made generous contributions to the wide range of software distributed through that system in reliance on consistent application of that system; it is up to the author/donor to decide if he or she is willing to distribute software through any other system and a licensee cannot impose that choice.

This section is intended to make thoroughly clear what is believed to be a consequence of the rest of this License.

8. If the distribution and/or use of the Program is restricted in certain countries either by patents or by copyrighted interfaces, the original copyright holder who places the Program under this License may add an explicit geographical distribution limitation excluding those countries, so that distribution is permitted only in or among countries not thus excluded. In such case, this License incorporates the limitation as if written in the body of this License.

9. The Free Software Foundation may publish revised and/or new versions of the General Public License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns.

Each version is given a distinguishing version number. If the Program specifies a version number of this License which applies to it and "any later version", you have the option of following the terms and conditions either of that version or of any later version published by the Free Software Foundation. If the Program does not specify a version number of this License, you may choose any version ever published by the Free Software Foundation.

10. If you wish to incorporate parts of the Program into other free programs whose distribution conditions are different, write to the author to ask for permission. For software which is copyrighted by the Free Software Foundation, write to the Free Software Foundation; we sometimes make exceptions for this. Our decision will be guided by the two goals of preserving the free status of all derivatives of our free software and of promoting the sharing and reuse of software generally.

#### NO WARRANTY

11. BECAUSE THE PROGRAM IS LICENSED FREE OF CHARGE, THERE IS NO WARRANTY FOR THE PROGRAM, TO THE EXTENT PERMITTED BY APPLICABLE LAW. EXCEPT WHEN OTHERWISE STATED IN WRITING THE COPYRIGHT HOLDERS AND/OR OTHER PARTIES PROVIDE THE PROGRAM "AS IS" WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESSED OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THE PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

12. IN NO EVENT UNLESS REQUIRED BY APPLICABLE LAW OR AGREED TO IN WRITING WILL ANY COPYRIGHT HOLDER, OR ANY OTHER PARTY WHO MAY MODIFY AND/OR REDISTRIBUTE THE PROGRAM AS PERMITTED ABOVE, BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY GENERAL, SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE THE PROGRAM (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY YOU OR THIRD PARTIES OR A FAILURE OF THE PROGRAM TO OPERATE WITH ANY OTHER PROGRAMS), EVEN IF SUCH HOLDER OR OTHER PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

#### END OF TERMS AND CONDITIONS

#### How to Apply These Terms to Your New Programs

If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.

To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively



convey the exclusion of warranty; and each file should have at least the "copyright" line and a pointer to where the full notice is found.

```
<one line to give the program's name and a brief idea of what it does.>
Copyright (C) 19yy <name of author>
```

This program is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version.

This program is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

You should have received a copy of the GNU General Public License along with this program; if not, write to the Free Software Foundation, Inc., 59 Temple Place, Suite 330, Boston, MA 02111-1307 USA

Also add information on how to contact you by electronic and paper mail.

If the program is interactive, make it output a short notice like this when it starts in an interactive mode:

```
Gnomovision version 69, Copyright (C) 19yy name of author
Gnomovision comes with ABSOLUTELY NO WARRANTY; for details type 'show w'.
This is free software, and you are welcome to redistribute it
under certain conditions; type 'show c' for details.
```

The hypothetical commands 'show w' and 'show c' should show the appropriate parts of the General Public License. Of course, the commands you use may be called something other than 'show w' and 'show c'; they could even be mouse-clicks or menu items--whatever suits your program.

You should also get your employer (if you work as a programmer) or your school, if any, to sign a "copyright disclaimer" for the program, if necessary. Here is a sample; alter the names:

Yoyodyne, Inc., hereby disclaims all copyright interest in the program 'Gnomovision' (which makes passes at compilers) written by James Hacker.

<signature of Ty Coon>, 1 April 1989  
Ty Coon, President of Vice

This General Public License does not permit incorporating your program into proprietary programs. If your program is a subroutine library, you may consider it more useful to permit linking proprietary applications with the library. If this is what you want to do, use the GNU Library General Public License instead of this License.

## References

- [Bac98] L. Bachmair. Personal communication at CADE-15, Lindau. Unpublished, 1998.
- [BDP89] L. Bachmair, N. Dershowitz, and D.A. Plaisted. Completion Without Failure. In H. Ait-Kaci and M. Nivat, editors, *Resolution of Equations in Algebraic Structures*, volume 2, pages 1–30. Academic Press, 1989.
- [BG94] L. Bachmair and H. Ganzinger. Rewrite-Based Equational Theorem Proving with Selection and Simplification. *Journal of Logic and Computation*, 3(4):217–247, 1994.
- [CL73] C. Chang and R.C. Lee. *Symbolic Logic and Mechanical Theorem Proving*. Computer Science and Applied Mathematics. Academic Press, 1973.
- [Der91] N. Dershowitz. Ordering-Based Strategies for Horn Clauses. In J. Mylopoulos, editor, *Proc. of the 12th IJCAI, Sydney*, volume 1, pages 118–124. Morgan Kaufmann, 1991.
- [DKS97] J. Denzinger, M. Kronenburg, and S. Schulz. DISCOUNT: A Distributed and Learning Equational Prover. *Journal of Automated Reasoning*, 18(2):189–198, 1997. Special Issue on the CADE 13 ATP System Competition.
- [DS94a] J. Denzinger and S. Schulz. Analysis and Representation of Equational Proofs Generated by a Distributed Completion Based Proof System. Seki-Report SR-94-05, Universität Kaiserslautern, 1994.
- [DS94b] J. Denzinger and S. Schulz. Recording, Analyzing and Presenting Distributed Deduction Processes. In H. Hong, editor, *Proc. 1st PASCO, Hagenberg/Linz*, volume 5 of *Lecture Notes Series in Computing*, pages 114–123, Singapore, 1994. World Scientific Publishing.
- [DS96] J. Denzinger and S. Schulz. Recording and Analysing Knowledge-Based Distributed Deduction Processes. *Journal of Symbolic Computation*, 21(4/5):523–541, 1996.
- [HBF96] Th. Hillenbrand, A. Buch, and R. Fettig. On Gaining Efficiency in Completion-Based Theorem Proving. In H. Ganzinger, editor, *Proc. of the 7th RTA, New Brunswick*, volume 1103 of *LNCS*, pages 432–435. Springer, 1996.

- [HJL99] Th. Hillenbrand, A. Jaeger, and B. Löchner. System Abstract: Waldmeister – Improvements in Performance and Ease of Use. In H. Ganzinger, editor, *Proc. of the 16th CADE, Trento*, volume 1632 of *LNAI*, pages 232–236. Springer, 1999.
- [McC94] W.W. McCune. Otter 3.0 Reference Manual and Guide. Technical Report ANL-94/6, Argonne National Laboratory, 1994.
- [MIL<sup>+</sup>97] M. Moser, O. Ibens, R. Letz, J. Steinbach, C. Goller, J. Schumann, and K. Mayr. SETHEO and E-SETHO – The CADE-13 Systems. *Journal of Automated Reasoning*, 18(2):237–246, 1997. Special Issue on the CADE 13 ATP System Competition.
- [MW97] W.W. McCune and L. Wos. Otter: The CADE-13 Competition Incarnations. *Journal of Automated Reasoning*, 18(2):211–220, 1997. Special Issue on the CADE 13 ATP System Competition.
- [NN93] P. Nivela and R. Nieuwenhuis. Saturation of First-Order (Constrained) Clauses with the Saturate System. In C. Kirchner, editor, *Proc. of the 5th RTA, Montreal*, volume 690 of *LNCS*, pages 436–440. Springer, 1993.
- [RV01] A. Riazanov and A. Voronkov. Vampire 1.1 (System Description). In R. Goré, A. Leitsch, and T. Nipkow, editors, *Proc. of the 1st IJCAR, Siena*, volume 2083 of *LNAI*, pages 376–380. Springer, 2001.
- [RV02] A. Riazanov and A. Voronkov. The Design and Implementation of VAMPIRE. *Journal of AI Communications*, 15(2/3):91–110, 2002.
- [Sch99] S. Schulz. The E Web Site. [http://www4.informatik.tu-muenchen.de/~\\$sim\\$schulz/~WORK/~eprover.h%tml](http://www4.informatik.tu-muenchen.de/~$sim$schulz/~WORK/~eprover.h%tml), 1999.
- [Sch00] S. Schulz. *Learning Search Control Knowledge for Equational Deduction*. Number 230 in DISKI. Akademische Verlagsgesellschaft Aka GmbH Berlin, 2000. Ph.D. Thesis, Fakultät für Informatik, Technische Universität München.
- [Sch01] S. Schulz. Learning Search Control Knowledge for Equational Theorem Proving. In F. Baader, G. Brewka, and T. Eiter, editors, *Proc. of the Joint German/Austrian Conference on Artificial Intelligence (KI-2001)*, volume 2174 of *LNAI*, pages 320–334. Springer, 2001.
- [Sch02] S. Schulz. E – A Brainiac Theorem Prover. *Journal of AI Communications*, 15(2/3):111–126, 2002.
- [Sch04] S. Schulz. System Description: E 0.81. In D. Basin and M. Rusinowitch, editors, *Proc. of the 2nd IJCAR, Cork, Ireland*, volume 3097 of *LNAI*, pages 223–228. Springer, 2004.

- [SSCG06] Geoff Sutcliffe, Stephan Schulz, Koen Claessen, and Allen Van Gelder. Using the TPTP Language for Writing Derivations and Finite Interpretations . In Ulrich Fuhrbach and Natarajan Shankar, editors, *Proc. of the 3rd IJCAR, Seattle*, volume 4130 of *LNAI*, pages 67–81, 4130, 2006. Springer.
- [Sut05] G. Sutcliffe. The TPTP Web Site. <http://www.tptp.org>, 2004–2005.
- [Tam97] T. Tammet. Gandalf. *Journal of Automated Reasoning*, 18(2):199–204, 1997. Special Issue on the CADE 13 ATP System Competition.
- [WAB<sup>+</sup>99] C. Weidenbach, B. Afshordel, U. Brahm, C. Cohrs, T. Engel, G. Jung, E. Keen, C. Theobalt, and D. Topic. System Abstract: SPASS Version 1.0.0. In H. Ganzinger, editor, *Proc. of the 16th CADE, Trento*, volume 1632 of *LNAI*, pages 378–382. Springer, 1999.
- [Wei99] C. Weidenbach. Personal communication at CADE-16, Trento. Unpublished, 1999.
- [Wei01] C. Weidenbach. SPASS: Combining Superposition, Sorts and Splitting. In A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, volume II, chapter 27, pages 1965–2013. Elsevier Science and MIT Press, 2001.
- [WGR96] C. Weidenbach, B. Gaede, and G. Rock. SPASS & FLOTTER Version 0.42. In M.A. McRobbie and J.K. Slaney, editors, *Proc. of the 13th CADE, New Brunswick*, volume 1104 of *LNAI*, pages 141–145. Springer, 1996.